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PHYSICS OF ROCKETS*

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By

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ELEMENTARY FACTS

A rocket is a jet-propelled device which carries within itself all the means for creating its propulsive jet. It differs, therefore, from the usual jet air-craft which require the oxygen of the air for burning fuel. Such jet craft could not operate in a vacuum, whereas a rocket can. In fact, a rocket performs most efficiently in a vacuum.

The principle of the rocket is found in the law of conservation of momentum. It should be recalled that the total momentum of a specified physical system remains unchanged unless some external force acts upon the system. The distribution of momentum between and among various parts of the given system may vary quite markedly even in the absence of any external force upon the system; but the sum total of the individual momenta of all the parts of the system nevertheless remains unchanged.

To understand how a rocket works, one must know clearly the meanings of mass, energy, and momentum. It will be assumed that the reader knows these concepts. In

*(Lecture at University of Wyoming, Institute of Chemistry and Physics, August, 1961)

Lecture presented at Wyoming U., Inst. of Chemistry and Physics, Aug. 1961 particular, it is assumed that the reader is familiar with the ideas of gravitational mass and inertial mass, and with the distinction between mass and weight.

Since kinetic energy is given by $\frac{1}{2}mv^2$, where m denotes mass and v speed, kinetic energy depends on the reference system used. This is so because speed is measured relative to a chosen reference frame. Thus, a body at rest on the earth has zero kinetic energy relative to the earth. But relative to the sun, the body has considerable kinetic energy due to the relative motion of the earth.

Momentum is given by $m\underline{v}$, where $\underline{v}$ is velocity. The vector character of velocity is indicated by underlining the letter used to denote it. Momentum is also a relative quantity, depending on the reference frame used.

It is important to keep the relative natures of energy and momentum in mind, and to be consistent in the use of the chosen frame of reference throughout a discussion or problem. Otherwise serious errors may result.

A SIMPLIFIED ROCKET

Suppose the motor of an idealized sort of rocket (Fig. 1) to consist of a cylindrical container from which a slug of mass m can be expelled by a spring. Let M be the mass of the rocket and let its initial velocity be V_o.

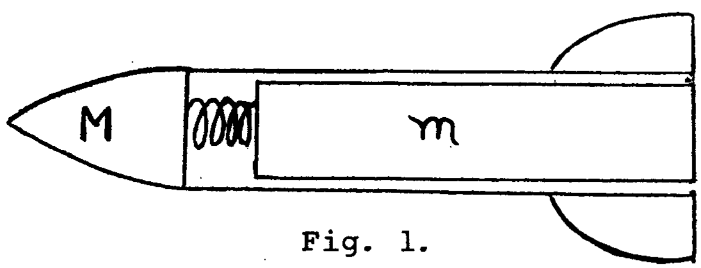

Fig. 1.

Then the total momentum of the rocket plus propellant mass is:

$$(M + m)\ V_o \tag{1}$$

and this must remain constant.

Now let the spring be released, expelling m at a speed c relative to M and opposite to V_o in direction. The mass M will now have a new speed V, making the total momentum of the system:

$$M\ V\ +\ m\ (V_o - c) \tag{2}$$

Equate (1) and (2) and solve for V:

$$M\ V + m\ (V_o - c) = (M + m)\ V_o$$

$$V = V_o + \frac{m}{M}\ c \tag{3}$$

Thus the velocity of M is increased by an amount which does not depend on the initial speed of M, but does depend on the ratio of propellant mass to the rocket mass, and upon the speed c with which m is expelled, increasing directly with both.

The rocket is accelerated by the simple expedient of throwing away mass. This is the principle of the rocket.

The above discussion represents the theoretical basis for a crude type of rocket sled. Imagine a sled on a smooth lake of ice. The rider has with him on the sled a big pile of heavy stones. By casting stones to the rear the sled can be made to accelerate forward. By casting stones forward, the sled can be slowed down again and stopped.

JETS

In an actual rocket, the propellant mass is expelled continuously in the hot gases of the jet, instead of all at once as in the idealized rocket described above. Thus, the rocket is a jet-propelled device.

A jet is a stream of fluid with a marked directional flow. Examples: water from a hose; flame of a blowtorch; air from a released toy balloon.

It takes a force to propel the fluid into the jet. By Newton's third law, the jet fluid reacts with an opposite and equal force on the container from which the jet issues. Thus, the jet exerts a thrust on the container, making it move.

There are many types of jet-propelled devices. Examples: Octopus. Turbojet airplane. Ramjet missile. Rockets. Only the rockets carry along all the materials needed to make the jet. Others rely on their surroundings for some of the material that goes into the jet. Hence, only the rocket can operate in the vacuum (or near vacuum) of outer space.

Strength of jet thrust depends on two important quantities: (1) speed of the fluid, and (2) size of the jet. Experimental check: For given size hoze, reaction from jet increases with the speed of water. For given water speed, reaction increases with size of hose.

In the case of very high temperature, high pressure gases, it has been found that supersonic jets can be obtained using a nozzle shaped as in Fig. 2, in which the

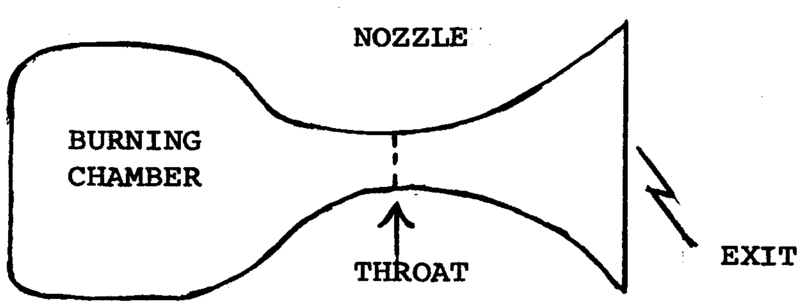

Fig. 2

gas channel first converges to a narrowest section, called the throat, and then diverges to the exit. Since high jet exhaust speed is so very important, all modern rockets use this type of nozzle. It is called a de Laval (or Laval) nozzle after a famous Swedish engineer of the 19th century.

THE ROCKET IN FREE SPACE

Let us consider the simple case of a rocket moving in a vacuum, away from the influence of either gravity or the atmosphere.

Let the velocity of the rocket be v; and its mass, m. As the propellants are consumed, m decreases from its initial value m_o to a "burnout" value m_b. If the jet exerts a thrust F upon the rocket, Newton's law gives:

$$m\dot{v} = F \tag{4}$$

Suppose the jet gases to be expelled at speed c, called the exhaust velocity. Now propellants are consumed at a rate $-\dot{m}$, where the dot denotes differentiation with respect to time. The minus sign is required, since m decreases as propellants are used, hence $\dot{m}$ is negative. Thus, at a given instant, $-\dot{m}$ of mass per second is going into the jet with a velocity c. This quantity of mass carries with it $-\dot{m}c$ of momentum. By the same sort of reasoning as that applied to the idealized rocket, the rocket must experience and equal change of momentum, but in the opposite direction. But, the rocket's instantaneous rate of change of momentum is given by $m\dot{v}$. Hence,

$$m\dot{v} = -\dot{m}c \tag{5}$$

This is the equation of motion of the rocket in free space. Comparing (5) with (4),

$$F = -\dot{m}c \tag{6}$$

which bears out the earlier statement that jet thrust varies directly with jet exhaust speed and quantity of material expelled per unit time.

Integrating (5) gives

$$v_b = v_o + c \ln R \tag{7}$$

where v_o is the initial velocity, if any, and where R is the ratio of the mass of the rocket at takeoff to the mass of the empty rocket at burnout. Thus, if m_e is the mass of the empty rocket, and m_p is the propellant mass, then

$$R = \frac{m_e + m_p}{m_e} \tag{8}$$

R is called the mass ratio of the rocket.

The importance of c and R as figures of merit for a rocket is clear. The exhaust velocity is increased by improving the power plant. High values for R are obtained by careful structural design.

The V-2 and Aerobee-Hi rockets have an exhaust velocity of 6400 feet per second. For Viking, c was 6200 feet per second. By present day standards, these are relatively poor. Since the V-2 days, typical exhaust velocities have risen to between 7000 and 9000 feet per second. The theoretical maximum for ordinary chemical rockets is about 13,000 feet per second. This would be obtained by burning liquid hydrogen with liquid oxygen or liquid fluorine.

A rocket with a mass ratio $R = 1$, would have no room for propellant. Thus, it couldn't get off the ground. One with a mass ratio of 2, would have its mass equally divided between structure and payload. This would be a very poor design. V-2, Aerobee-Hi, and Viking 11 had mass ratios 4.0, 4.8, and 6.2 respectively. Good design today produces ratios in the neighborhood of 10 or higher. A mass ratio $R = 20$ implies that only 5% of the total mass is structure. It is questionable whether such a structure could take the weights of propellants and other forces generated during powered flight.

From (6) it is seen that c is given by the ratio of the rocket thrust to the rate at which propellants are consumed. If the total time of burning is T, one can also write

$$c = \frac{FT}{-\dot{m}T} = \frac{I}{m_p} \tag{9}$$

where I is the total impulse delivered to the rocket, and m_p is the total propellant mass consumed. Engineers prefer to talk about propellant weights, and use the quantity I_{sp} in place of c as a figure of merit, where

$$I_{sp} = \frac{c}{g_o} = \frac{I}{m_p g_o} = \frac{I}{w_p} \tag{10}$$

g_o being the acceleration of gravity at sea level, and w_p being the weight of the propellants. The quantity I_{sp} is called the specific impulse of the rocket motor. With a consistent set of units, I_{sp} comes out in units of time. It is left to the reader to convert the values of c discussed above, to equivalent specific impulses in seconds.

Power is work done per unit time. Since work is force times distance, power can be written, when convenient to do so, as force times velocity. Thus, for a rocket one can express the power P being expended as

$$P = Fv$$

but since v varies from zero at the start to great velocities at burnout, this parameter is not especially useful in connection with rockets. For the V-2 the thrust was 28 tons and the burnout velocity on the order of one mile per second. Thus, the power generated varied from zero at start to about half a million horsepower near burnout.

Another mass ratio in common use is

$$\zeta = \frac{m_p}{m_e + m_p}$$

the ratio of the propellant mass to the total mass at takeoff. In term of ζ, R is

$$R = \frac{1}{1 - \zeta} .$$

The reader may convert the values of R previously discussed, to equivalent values of ζ.

THE STEP ROCKET IN FREE SPACE

Mass ratio R is increased by eliminating structural mass, while keeping the same propellant load. This fact permits one to improve the performance of a rocket by what is called multiple staging. In this process first one rocket propels one or more other rockets. After the first rocket is burned out it is discarded, and a second rocket takes over. When the second one is used it may be discarded, and a third stage ignited. And so on. The effect is a continuing improvement of mass ratio, with a resultant improvement in performance.

Let us consider a simple case. Let the step rocket be as shown in Fig. 3.

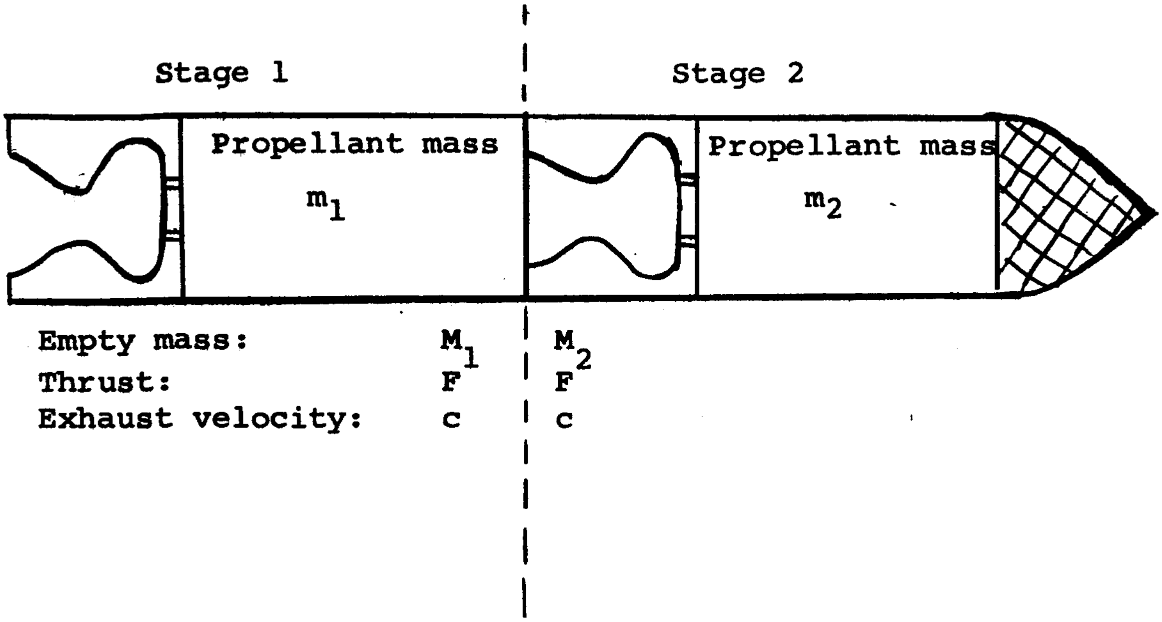

Fig. 3

Using (7) the velocity v_{b1} at the burnout of the first stage would be:

$$v_{b1} = c \ln \frac{M_1 + M_2 + m_1 + m_2}{M_1 + M_2 + m_2}$$

assuming start from rest. Letting

$$R_1 = \frac{M_1 + M_2 + m_1 + m_2}{M_1 + M_2 + m_2}$$

$$v_{b1} = c \ln R_1$$

Using (7) again, after discarding the empty first stage, the velocity at second stage burnout would be

$$v_{b2} = c \ln R_1 + c \ln \frac{M_2 + m_2}{M_2}$$

$$= c \ln R_1 + c \ln R_2$$

where $R_2 = (M_2 + m_2)/M_2$. Combining logarithms

$$v_{b2} = c \ln R_1 R_2$$

Thus, since $R_2 > 1$, the effect of double staging is to achieve a performance equivalent to that of a rocket with mass ratio greater than R_1. Actually the rocket shown in the figure is of poor design. It does not make best use of the staging technique. As the reader can show with a few numerical examples, it is better to have the second stage appreciably smaller than the first stage, so that R_1 can be large as well as R_2.

THE ROCKET IN THE EARTH'S GRAVITATIONAL FIELD AND ATMOSPHERE

The previous discussion led to some general conclusions about rocket performance. The discussion, however, was very greatly simplified by assuming free space conditions. For a rocket moving in the vicinity of the earth the earlier conclusions still hold in a qualitative way, but quantitatively are considerably modified.

The differential equation of motion now takes the form:

$$m \, \underline{\dot{v}} = \underline{F} + m \, \underline{g} + \underline{D} \tag{11}$$

where the underlined quantities are vectors, having direction as well as magnitude.

The vector $\underline{F}$ gives the thrust generated by the jet. Its magnitude is still given principally by (6) but the presence of the atmosphere causes a modification which can vary by several percent between sea level and high altitude conditions. As a general rule F is essentially constant, especially in liquid propellant rockets. In some solid propellant rockets, F varies considerably during burning.

The vector $\underline{g}$ is the acceleration of gravity, which varies both in direction and magnitude. Exdept for higher order effects its direction is toward the center of the earth. Its magnitude follows the inverse square law. In rocket calculations $\underline{g}$ may be handled as indicated in the table below:

Situation	$\underline{g}$
Limited heights and ranges	Constant magnitude Constant direction *
Appreciable height changes but limited ranges	Magnitude = $g_o \left(\frac{a}{a+h}\right)^2$ † Constant direction *
Appreciable changes in both height and range	Magnitude = $g_o \left(\frac{a}{a+h}\right)^2$ † Variable direction *

* Toward center of earth.

† g_o = acceleration of gravity at surface of earth

a = radius of earth

h = height

The vector $\underline{D}$ is the drag on the rocket due to the presence of the atmosphere. It is always opposite to the motion through the air, in the direction of what is called the relative wind. The relative wind results from a vectorial combination of the rocket's absolute velocity with any actual atmospheric wind. Although the induced wind is usually thought of as the principal part of the relative wind, quite often the effect of a true wind is not negligible.

The magnitude of $\underline{D}$ is given by

(12) $$D \doteq \tfrac{1}{2} \rho V^2 C_D A$$

where ρ is the air density, V the relative wind speed, A the rocket's cross section area normal to the relative wind, and C_D the drag coefficient. It is common to measure C_D in wind tunnel tests on models. For order of magnitude calculations, however, one may set $C_D = 1$. Thus, a sphere of radius 0.3 meters, moving at 1000 meters per second through the air at 30 kilometers altitude, would experience a drag of approximately

$$\tfrac{1}{2}\left(2 \times 10^{-5} \ \frac{\text{gm}}{\text{cm}^3}\right)\left(10^5 \ \frac{\text{cm}}{\text{sec}}\right)^2 \cdot 1 \cdot \left(\pi\, 3^2 \times 10^2 \ \text{cm}^2\right)$$

$= 2.8 \times 10^8$ dynes.

$\cong$ 600 pounds.

In the most general case, the directions of $\underline{v}$, $\underline{F}$, $\underline{g}$, $\underline{D}$ in (11) may all be different. The integration of this equation is often very involved and difficult. For our purposes, however, suppose we take the simple case of a vertical sounding rocket, neglecting the atmosphere.

THE VERTICAL SOUNDING ROCKET NEGLECTING DRAG

In this case the equation (11) reduces to

$$m \dot{v} = - \dot{m} c - m g$$

(13) $$\dot{v} = - \frac{\dot{m}}{m} c - g$$

Assuming $\dot{m}$, c, and g all constant, (13) can be integrated to give burnout velocity and the peak altitude reached by the rocket. The burnout velocity is:

(14) $$v_b = c \ln R - g t_b,$$

where t_b is the burning time. Thus, there is a velocity penalty caused by the presence of gravity. The penalty is decreased by decreasing the burning time. In actuality, however, t_b cannot be reduced too far, since the increased speeds attained in the lower atmosphere then incur a drag penalty due to air friction which we have neglected here. Also the acceleration becomes intolerably great.

The maximum altitude attainable is given by a second integration:

(15) $$h_{max} = \frac{c^2 \ln^2 R}{2g} - c t_b f(R)$$

where

$$f(R) = \ln R - 1 + \frac{\ln R}{R - 1}$$

The quantity $f(R)$ is positive for $R > 1$ and increases with R.

The principal term on the right of (15) is the first one, which shows that the peak altitude of a sounding rocket increases with the square of the exhaust velocity (or specific impulse) and as the square of the natural logarithm of the mass ratio. Once again there is a penalty term which, in this case in which air resistance is ignored, decreases as the burning time decreases.

PROPELLANTS

- Chemical propellants
  - Solids
    - Double base
    - Composites
  - Liquids
    - Monopropellants
    - Bipropellants
  - Free radicals
- Nuclear propulsion
  - Direct use of nuclear particles
  - Heating and ejecting working fluid
  - Conversion to electrical power
- Electrical propulsion
  - Ion
  - Plasma
- Solar propulsion
  - Conversion to electrical power
    - Direct
    - By means of generators
  - Sailing
- Photon propulsion

GENERATION OF THE JET

Let us try to visualize what happens in a rocket jet. Let the throat area of the nozzle be A_t and the exit area, A_e. The ratio A_t/A_e is called the expansion ratio for the nozzle. Suppose, now, that the combustion is started in the burning chamber. As the pressure p_o in the chamber increases, a flow in the nozzle will take place. At first, the flow will be entirely subsonic and analogous to that of an incompressible fluid in a venturi tube, with increasing velocities in the convergent portion and decreasing velocities in the divergent section of the nozzle. When p_o reaches a certain critical value, considerably above the ambient pressure p_a outside the rocket, the flow will become sonic at the throat. For all higher values of p_o the jet will be of sonic speed at the throat and supersonic in the divergent section. The flow upstream from the throat is then entirely determined by conditions in the combustion chamber. For example, once this supersonic nozzling is achieved, the mass flow rate $-\dot{m}$ is completely determined by the throat area A_e and combustion-chamber conditions.

The exhaust velocity v_e of the jet will be that existing at the nozzle exit and is the value to use when calculating thrust. If the jet exhausts into a vacuum, the pressure p_e at the nozzle exit, and the exhaust velocity v_e, will be determined by the combustion chamber conditions and the nozzle expansion ratio. There is little change in p_e and v_e for ambient atmospheric pressures p_a lower than p_e. When $p_a > p_e$, the jet establishes a pressure match with the outside atmosphere by means of a series of shock waves in the divergent section of its flow, with a resultant loss in exhaust velocity. If the expansion ratio is such that $p_e = p_a$, it can be shown that for each set of combustion-chamber conditions the total thrust is the maximum obtainable with the nozzle in question. Under these conditions, the nozzle is said to be perfectly matched to the ambient atmosphere. For a vertical sounding rocket, it is obviously not possible to match the rocket nozzle to all pressures encountered, and it is customary to use expansion ratios that achieve a match at a suitable altitude between sea level and the highest altitude reached during powered flight.

CONSTRUCTION OF A ROCKET

Layout

Materials

Shapes

GUIDANCE AND CONTROL

Methods of control

Types of guidance

Unguided rockets. Weathercocking. Wind computations.

Inertial guidance.

Line of sight.

Beam riders.

Celestial naviagation.

Radio grids.

USES OF ROCKETS

Fireworks

Signalling

Sea rescue operations

Aircraft boost

Aircraft propulsion (e.g., X-15)

Weapons.

Sounding rockets.

Space vehicles.

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